# MathSketch: Designing a Dynamic Whiteboard for Instructional Contexts


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## Abstract
Traditional whiteboards are a common medium for mathematics education, and are particularly suited to the high school and college level where conceptual understanding of the subject matter is emphasized above procedural understanding. Although considerable work has been done to apply sketch-based interaction to mathematics learning, very few have addressed this from the perspective of teaching mathematics in a conventional classroom environment. We provide a set of design considerations for dynamic whiteboards in instructional contexts and present an embodiment of these considerations in our prototype, MathSketch.



## Author Keywords
Electronic whiteboards, mathematics, education, sketching, representation

## ACM Classification Keywords
H.5.m. Information interfaces and presentation (e.g., HCI): Miscellaneous.


## Introduction
In spite of the growing body of technologies available for instruction, traditional whiteboards remain a cornerstone in mathematics education. Researchers studying the educational use of whiteboards report that they provide several affordances that instructors

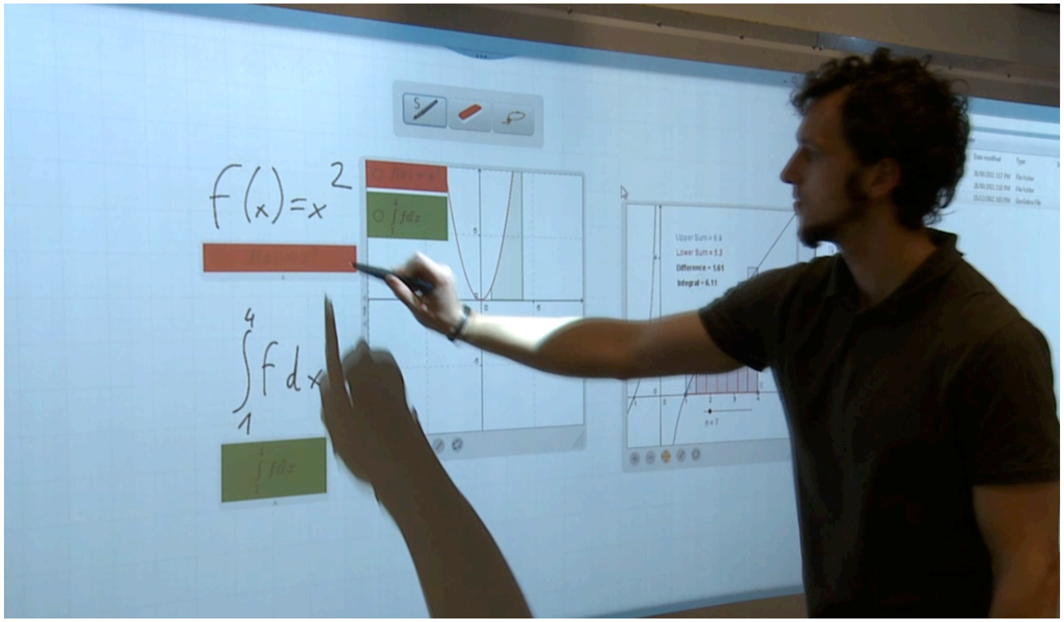

**Figure 1.** MathSketch is designed from ground-up as an instructional tool for interactive whiteboards. It relies on novel design conventions, accounting for both instructors' and learners' needs.

employ in their everyday practice. For example, Lanir et al. argue that, in contrast to slideware-based tools, whiteboards offer: non-linear explanation, complex reasoning, temporally gradual build-up of information, and juxtaposition of multiple representations (e.g. text, diagram, table, graph). These practices are particularly important in the context of senior high-school and early college-level mathematics, where the focus moves from concrete, algebraic manipulations (i.e. procedural understanding), to more abstract mathematical thinking (i.e. conceptual understanding). Here, the whiteboard allows for progressive revealing and temporal build-up of information, which are characteristic of mathematics instruction at this level. Furthermore, the multiple representations play a fundamental role in reaching students that learn differently [1,5].

Our goal is to design tools that support mathematics instruction with computation that meet the needs of both instructors and students. Here, we present MathSketch (Figure 1), which illustrates our approach. We begin with whiteboard-style instruction, exploring sketch-based mathematics systems in this context. Our intention is to provide interaction strictly through pen-based interaction while supporting a sophisticated underlying mathematics implementation to provide fluid interaction. These interactions allow for multiple-line mathematics equations, and provide multiple representations of mathematics formulae (e.g. graphs). As a useful side-effect, we provide mechanisms for learners to interactively manipulate equations and generated visualizations. While prior work has explored sketch-based interactions for mathematics, most have been designed for learners on tablet-sized form factors. As our primary interest is to support mathematics instruction, the instructional context of a teacher imposes a very different set of design requirements that significantly impact the design.

Although a considerable body of work has considered sketch-based interaction for mathematics, very little of this work has used teaching/instruction as a starting point. While pragmatically, many systems designed for tablets can be simply projected onto a whiteboard, the fluidity of instruction may be severely limited. We have seen from the interactive whiteboard literature that the social dynamics of these situations demand different interaction mechanisms. For example, that interactions need to be visible and understandable so that viewers can interpret and learn how to use the system.

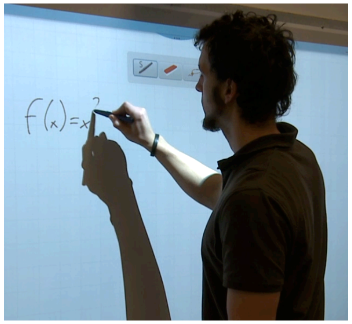

**Figure 2.** Hand-writing an equation.

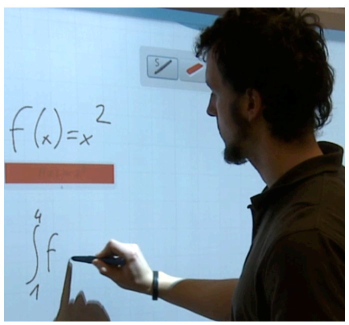

**Figure 3.** The system recognizes the equation.

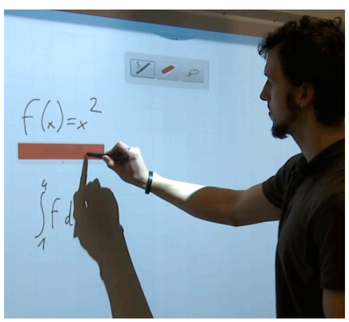

**Figure 4**. Tapping the recognized equation results in a graph.

Generally, interactive math tools have not been tailored to teaching scenarios. In our work here, we aim to address this gap.

## Related Work

Considerable research and commercial effort has gone into building computational support for mathematics. Pertinent to our work here, we outline three major themes from these works: symbolic manipulation, sketch-based interaction, and instructional aids for mathematics instruction.

*Symbolic manipulation*. While computers were originally designed to compute numerical quantities represented by various symbols, a significant advance in mathematical computing was the inclusion/design of symbol manipulation techniques. This allows for traditional symbolic analysis, and the manipulation of math equations in symbolic form rather than approximations. Several commercial systems now support symbolic analysis of mathematical problems. The underlying infrastructure supporting this analysis, called computer algebra systems (CASs), form the core of many newer systems that support mathematics instruction.

*Sketch-based interaction*. Advances in pen-based computing led the development of gesture-recognizers capable of supporting sketch-based input for mathematics systems. These recognition systems provide a front-end to underlying CASs, and allow people to handwrite complex math expressions (e.g. [2,3,4,6,7,8]). Recent efforts (e.g. [5,8]) augment the sketch-based approach, employing *both* pen- and touch-input in a rich, fluid interface for mathematics content. In [5], the authors consider a virtual paper metaphor for displaying and solving high school-level mathematics problems. [8] considers this approach in a whiteboard-sized context.

*Alternate visual representations.* Several systems have also been designed for use on tablet form-factors, integrating sketching with visualization. MathJournal[1] and Microsoft Math[2] focus on solving equations, and providing visualizations to accompany math problems, such as graphs and charts. Similarly, MathPad [3] provides interactive visualizations, where dynamic drawings can be connected to mathematical expressions.

## Design Considerations

With due consideration to prior work, we developed a set of design requirements for mathematics teaching systems. The focus on teaching introduces three sets of related design considerations, as there are three different conceptual "users" of such systems: teachers, students in the audience, and students *using* the system to learn.

*Supporting existing teaching practices.* It is important to support teachers' existing practices with instructional aids, as there has been a general backlash toward using too much technology in the classroom. As mentioned, instructors already employ whiteboards in very particular ways when teaching mathematics.

*Support low-overhead, effortless interaction*. Using the system should feel natural for the teacher and needs to be as fluid as traditional teaching: that is to say, the system should not impose additional "work" to gain computational benefit. The teacher should be able to

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[1] http://www.xthink.com

[2] http://www.microsoft.com

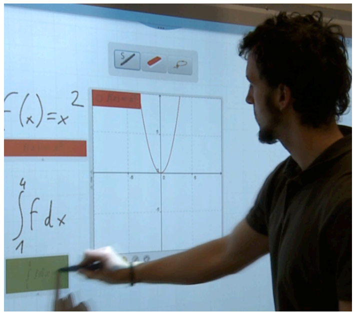

**Figure 5.** Grabbing another equation.

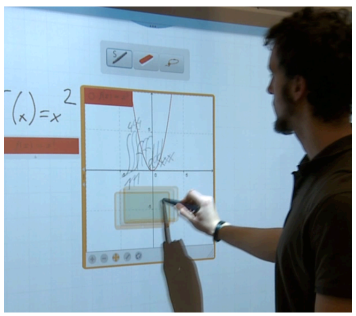

**Figure 6.** Dragging second equation to the graph.

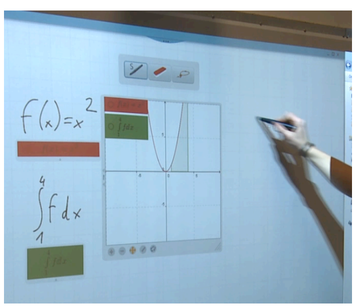

**Figure 7.** Graph illustrates both.

focus on processes in the classroom while using the system. The cognitive effort to put student's ideas into the system should be low.

*Account for the teaching context*. Traditional WIMP based systems are not appropriate. Switching between different tools and being forced to create content only at dedicated areas and in predefined order can break the fluidity of the interaction process. Triggers to execute commands should be accessible from any position in front of the whiteboard; therefore the use of a fixed menu bar is inappropriate. The interaction possibilities should always be displayed near the region they will affect.

## MathSketch Prototype

We have designed and built a prototype, MathSketch, that embodies our design considerations identified above. Due to space considerations, we illustrate this system primarily through a set of figures that illustrate some of the core functionality (Figures 2-7).

*Larry sketches an equation onto the whiteboard. The system reveals (in red) how it interpreted the handwriting. Tapping on the red-coloured box produces an alternate representation (a graph). This graph can be manipulated, but Larry then drags a second equation onto the original graph, which results in both graphs being displayed simultaneously.*

We are currently evaluating this prototype through a series of focused design sessions with mathematics instructors in an international study.


## Acknowledgements

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